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Laminar free-convection in water with variable physical properties adjacent to a vertical plate with uniform heat flux

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Abstract

The steady laminar boundary layer flow of water along a vertical stationary uniform flux plate is studied. The working fluid is water whose density–temperature relationship is nonlinear at low temperatures and viscosity and thermal conductivity are functions of temperature. The results are obtained with the numerical solution of the boundary layer equations and cover the temperature range between 40 and 0 °C taking into account the temperature dependence of μ, k and ρ. Both upward and downward flow is considered. The variation of μ, k and ρ with temperature has a strong influence on the results.

Keywords: Uniform flux plate; Free convection; Water; Nonlinearity

1. Introduction

The earliest known theoretical treatment of free convection along a vertical uniform flux plate is the analysis of Sparrow and Gregg [17]. There are numerous subsequent similar investigations in the literature. The reader can find many relevant works in a very recent paper by Aydin and Guessous [1]. In the present paper we focus on water whose density–temperature relationship is linear at high temperatures and nonlinear at low temperatures. The density of pure water is maximum at 3.98 °C. The density increases as the temperature decreases approaching 3.98 °C, while the density decreases as the temperature decreases from 3.98 to 0 °C (see Fig. 1). The density profile is approximately parabolic in the vicinity of 3.98 °C and there are infinite couples of temperatures near the density extremum with equal densities. For example, the density at 0 °C is equal to that of 8.13 °C. Some works relevant to water at high temperatures, where the density–temperature relationship is linear, are those by Goldstein and Eckert [6]; Qureshi and Gebhart [14]; King and Reible [8]; Inagaki and Komori [7] and Pittman et al. [13].

Except of the above mentioned studies there are some papers concerning water free convection in low temperature range where the density–temperature relationship is nonlinear. Soundalgekar [16] used an integral method to study water free convection over a vertical plate with variable temperature. The ambient water was 4 °C. Gebhart and Mollendorf [3] analyzed the problem of laminar free convection of water over a heated vertical plate and gave the appropriate conditions for similarity solution for the uniform heat flux case. The same problem was analyzed by Qureshi and Gebhart [15] and similarity results were produced only when the ambient density coincides with density extremum temperature. Gebhart et al. [4] developed a purturbation analysis to extend the range of the calculations beyond the density extremum temperature and produced results for the adiabatic and uniform heat flux plate. These results are valid only when the ambient temperature is near the maximum density temperature. Pantokratoras [10] presented results for the uniform flux plate in the temperature range between 20 and 0 °C taking into account the

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Nomenclature

f	dimensionless stream function
g	gravitational acceleration
Gr_x	local Grashof number, $Gr_x = \frac{gx^3}{v^2}\frac{\rho_a - \rho_o}{\rho_a}$
k	thermal conductivity
Nu_x	local Nusselt number, $Nu_x = \frac{qx}{k(T_o - T_a)}$
Pr	Prandtl number, $Pr = v/\alpha$
q	surface heat flux
Ra^*	modified Rayleigh number, $Ra^* = g\beta qx^4/v\alpha k$
T	water temperature
u	vertical velocity
v	horizontal velocity
x	vertical coordinate
y	horizontal coordinate

Greek symbols

α	thermal diffusivity
β	thermal expansion coefficient of water
n	similarity variable, $n = \frac{y}{x}\left[\frac{Gr_x}{4}\right]^{1/4}$
μ	dynamic viscosity
v	kinematic viscosity
ρ	water density
θ	dimensionless temperature, $\theta = \frac{T - T_a}{T_o - T_a}$

Subscripts

a	ambient
o	plate
f	film

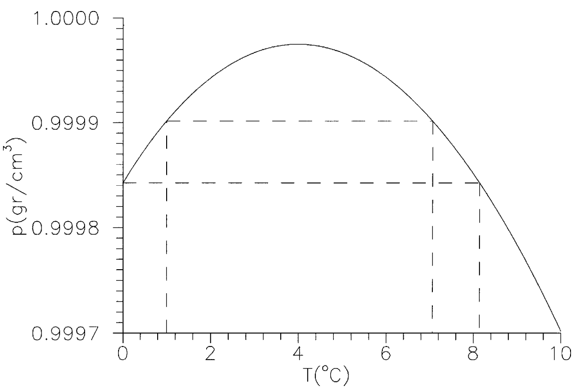

Fig. 1. Variation of water density in the 10–0 °C region.

water nonlinearity at low temperatures. The results of all the above works, concerning the nonlinear region, are valid only for small temperature differences between the plate and the ambient water (about 3 °C).

All the above mentioned works, either at the linear or the nonlinear region, assume constant dynamic viscosity and thermal conductivity both taken at ambient or film temperature. However, these quantities are also functions of temperature. In the temperature range between 40 and 0 °C the water kinematic viscosity varies from 0.005827 to 0.017911 cm^2/s and the Prandtl number varies from 3.83 to 13.18 [9]. The objective of the present paper is to present results for laminar free convection of water along a vertical uniform flux plate in the temperature range between 40 and 0 °C, taking into account the temperature dependence of all water physical properties (μ, k and ρ).

The boundary layer equations with variable fluid properties were solved directly, without any transformation, by a method described by Patankar [12]. The International Equation of State for Seawater [2] has been used for the calculation of density from temperature. For the calculation of dynamic viscosity and thermal conductivity the formulae given by Kukulka et al. [9] have been used. The finite difference method is used with primitive coordinates x, y and a space marching procedure is used in x direction with an expanding grid. The accuracy of the method was tested comparing the results with those of the classical free convection problem (constant viscosity and thermal conductivity and linear relationship between density and temperature). The comparison was satisfactory. More information about the equations and the solution procedure may be found in Pantokratoras [11]. The boundary conditions were as follows:

$$\text{at } y = 0: \quad u = v = 0, \quad -k\left[\frac{\partial T}{\partial y}\right] = q \tag{1}$$

$$\text{as } y \to \infty: \quad u = 0, \quad T = T_a \tag{2}$$

where q is the heat flux at the plate and T_a is the ambient water temperature.

2. Results and discussion

In the similarity method commonly used in free convection over vertical surfaces, the following functions and variables are used. The nondimensional stream function $f(n)$, the similarity variable n, the local Grashof Gr_x and the nondimensional velocity f'. These quantities are well known and can be found in the literature (see for example [11]). The most important quantities for this problem are the wall heat transfer and the wall shear stress defined as

$$\theta'(0) = \frac{x}{T_o - T_a}\left[\frac{Gr_x}{4}\right]^{-1/4}\left[\frac{\partial T}{\partial y}\right]_{y=0} \quad (3)$$

$$f''(0) = \frac{\rho_o x^2}{\mu_o\sqrt{2}}[Gr_x]^{-3/4}\left[\frac{\partial u}{\partial y}\right]_{y=0} \quad (4)$$

where T_o and T_a are the plate and ambient temperatures and ρ_o and μ_o are the water density and dynamic viscosity at the wall.

It is known from the literature that in the classical free convection with uniform surface flux the plate temperature increases along the plate according to the law

$$\Delta T = T_o - T_a = Nx^n \quad (5)$$

where the similarity exponent n is equal to $1/5$. In the present work results were produced for four ambient water temperatures ($T_a = 3.98$, 10, 20 and 30 °C). In each numerical experiment the plate temperature varied from the ambient temperature to 40 °C ($\Delta T = 36$, 30, 20 and 10 °C). In addition, except of the usual problem of the upward moving fluid, results have been produced for downward flow. This can be achieved by cooling the plate instead of heating it (q was taken negative in Eq. (1)). In this case the plate temperature decreases along the plate. For the downward flow results were produced again for four ambient water temperatures ($T_a = 40$, 30, 20 and 10 °C). In each numerical experiment now the plate temperature varied from the ambient temperature to 0 °C ($\Delta T = 40$, 30, 20 and 10 °C). The reason for taking as lowest ambient temperature 3.98 °C for the upward flow and 0 °C as final temperature for the downward flow is the following. The density of water at 3.98 °C is maximum and all temperatures greater than 3.98 °C correspond to densities lower than that of 3.98 °C, so we have pure upward flow at this temperature region. The water density at 0 °C and all densities at intermediate temperatures are greater than that of the ambient temperatures (40, 30, 20 and 10 °C) and thus the fluid sinks.

In each numerical experiment the local Grashof and the local Prandtl number have been considered variable along the flow. The solution procedure starts from the plate leading edge and marches in the vertical direction. At every downstream position we calculated the plate temperature and then the local Grashof and the local Prandtl number have been calculated at film temperature $(T_o + T_a)/2$ which also changes along the plate. The correspondence between the Prandtl number and the water temperature is as follows:

T (°C)	40	30	20	10	0
Pr	3.83	5.41	6.99	9.31	13.18

In Fig. 2 the wall heat transfer $-\theta'(0)$ is shown as function of the film Prandtl number Pr_f for different

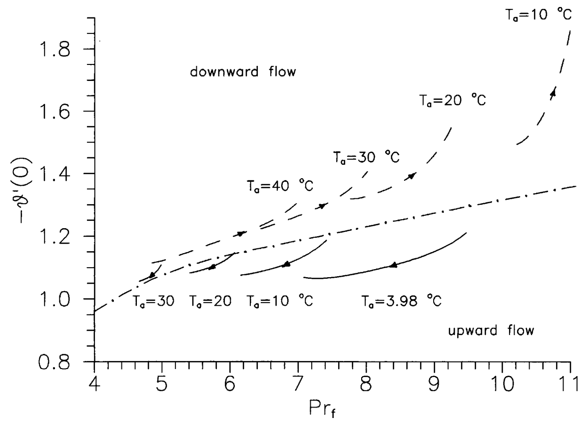

Fig. 2. Wall heat transfer as function of film Pr number for different ambient temperatures. Broken line with dots corresponds to constant properties. Solid lines correspond to upward flow and dashed lines to downward flow. Arrows show increasing ΔT.

ambient temperatures. The broken line with dots corresponds to heat transfer of the classical problem of free convection along a uniform flux vertical plate of a fluid with constant properties and linear density–temperature relationship. Values of $-\theta'(0)$ for the classical problem were taken from Gebhart [5]. In his work there is a table with results concerning the transport quantities of free convection adjacent to vertical isothermal and adiabatic surfaces. This table was prepared by Krishnamurthy. The solid lines correspond to upward flow and the dashed lines to downward flow. Arrows show increasing ΔT. From this figure it is seen that as ΔT increases along the plate the wall heat transfer $-\theta'(0)$ decreases in the upward flow and increases in the downward flow. It is remarkable that as we move from the nonlinear to linear region the curves approach the constant properties line.

In Fig. 3 the wall shear stress $f''(0)$ is shown for the same conditions of Fig. 2. Now the broken line with dots corresponds to wall shear stress of the classical problem of free convection along a uniform flux vertical plate of a fluid with constant properties. This line has been produced with values of $f''(0)$ taken again from Gebhart [5]. The solid lines correspond to upward flow and the dashed lines to downward flow. Arrows show increasing ΔT. From this figure it is seen that as ΔT increases the wall shear stress increases in the upward flow and decreases in the downward flow except of the case of downward flow between 10 and 0 °C. For this case the wall shear stress increases as ΔT increases in contrary to the other three cases of downward flow. It is should be noted that in the case of $T_a = 3.98$ °C (upward flow) the wall shear stress lies below the constant properties line for small ΔT and above this line for large ΔT.

Another interesting quantity calculated in the present work is the value of the exponent n in Eq. (5). In the

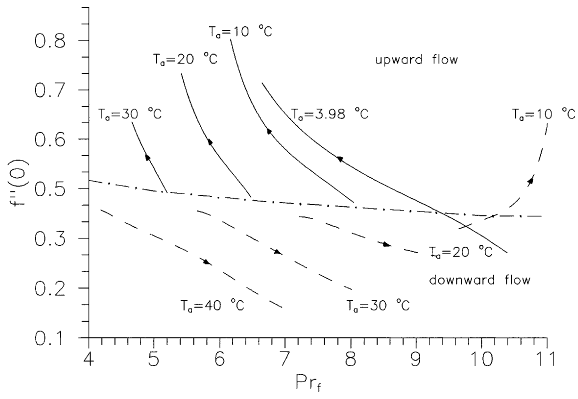

Fig. 3. Wall shear stress as function of film Pr number for different ambient temperatures. Broken line with dots corresponds to constant properties. Solid lines correspond to upward flow and dashed lines to downward flow. Arrows show increasing ΔT.

classical problem with constant properties and linear density–temperature relationship the value of n is 0.20. In the nonlinear region the value of n departs from the above value. In the following table the calculated values of n for different temperature ranges are given.

	Upward flow			
Temperature range °C	3.98 ⇒ 40	10 ⇒ 40	20 ⇒ 40	30 ⇒ 40
n	0.1649	0.1766	0.1890	0.2000
	Downward flow			
Temperature range °C	40 ⇒ 0	30 ⇒ 0	20 ⇒ 0	10 ⇒ 0
n	0.2281	0.2253	0.2203	0.2407

From the above table it is seen that the exponent n is lower than 0.20 in the upward flow and higher than 0.20 in the downward flow. The biggest deviation from the classical value occurs at the strongly nonlinear region between 10 and 0 °C.

Another interesting quantity in heat transfer problems is the local Nusselt number defined as

$$Nu_x = \frac{qx}{k_f(T_o - T_a)} \tag{6}$$

where k_f is the water thermal conductivity at film temperature. The most recent correlation of Nusselt number for free convection along a vertical plate with constant heat flux is that by Aydin and Guessous [1]. They proposed the following equation which correlates the local Nusselt number, Prandtl and the modified Rayleigh number

$$Nu_x = C_1\left(\frac{Ra^*Pr}{0.67 + Pr}\right)^m \tag{7}$$

where the exponent m is again 0.20 and the constant $C_1 = 0.630$. These values are valid for fluids with linear density–temperature relationship. In the present work the local Nusselt number has been calculated in each numerical experiment from Eq. (6) and the values have been compared with the results of Eq. (7). It should be noted here that the local modified Rayleigh number and the local Prandtl number, used in Eq. (7), have been considered again variable along the flow. At every downstream position we calculated the plate temperature and then the two numbers have been calculated at film temperature. The differences between our results and those by Aydin and Guessous [1] varied from 2% (temperature range 40 ⇒ 0, downward flow) to 17% (temperature range 3.98 ⇒ 40, upward flow). These differences are reasonable because Eq. (7) concerns fluids with linear density-temperature relationship. After that we tried to adjust the exponent m and the constant C_1 to our results. In the following table the calculated values of m and C_1 for different temperature ranges are given.

	Upward flow			
Temperature range °C	3.98 ⇒ 40	10 ⇒ 40	20 ⇒ 40	30 ⇒ 40
m	0.2000	0.1983	0.1984	0.1980
C_1	0.542	0.599	0.621	0.637
	Downward flow			
Temperature range °C	40 ⇒ 0	30 ⇒ 0	20 ⇒ 0	10 ⇒ 0
m	0.2006	0.2016	0.2039	0.2139
C_1	0.604	0.591	0.556	0.484

From the above table it is seen that the exponent m is slightly lower than 0.20 in the upward flow and slightly higher than 0.20 in the downward flow. The constant C_1 and the exponent m approach the values given by Aydin and Guessous [1] as we move from the nonlinear region to linear region. The differences between our results and those by Eq. (7) with the above modified C_1 and m values are below 1% for all temperature ranges.

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